Aufgaben zu Wurzeln

Lösungen

Wurzelziehen (Teil 1)

a) $\sqrt{121}=11$

b) $\sqrt{4a^2}=\sqrt{4}\cdot\sqrt{a^2}=2a$

c) $\sqrt{x^4}=\sqrt{(x^2)^2}=x^2$

d) $\sqrt{a^2b^2}=\sqrt{(a\cdot b)^2}=ab$

e) $\sqrt{n^2m^6}=\sqrt{n^2\cdot m^{3\cdot 2}}=\sqrt{n^2\cdot(m^3)^2}$
$=\sqrt{(n\cdot m^3)^2}=n\cdot m^3$

f) $\sqrt{a^3\cdot a^5}=\sqrt{a^{3+5}}=\sqrt{a^8}=\sqrt{a^{4\cdot 2}}=\sqrt{(a^4)^2}=a^4$

g) $\sqrt{\frac{49}{25}}=\frac{\sqrt{49}}{\sqrt{25}}=\frac{9}{5}$

h) $\sqrt{\frac{a^2}{81}}=\frac{\sqrt{a^2}}{\sqrt{81}}=\frac{a}{9}$

i) $n\neq 0:\ \sqrt{\frac{36n}{n^3}}=\sqrt{\frac{36}{n^2}}=\frac{\sqrt{36}}{\sqrt{n^2}}=\frac{6}{n}$

j) $\sqrt{x^2+2xy+y^2}=\sqrt{(x+y)^2}=x+y$

k) $n\leq 3:\ \sqrt{9-6n+n^2}=\sqrt{(3-n)^2}=3-n$

l) $a+b\neq 0:\ \sqrt{\frac{(a^2-b^2)\cdot(a-b)}{a+b}}$
$=\sqrt{\frac{(a+b)\cdot(a-b)\cdot(a-b)}{a+b}}$
$=\sqrt{(a-b)^2}=a-b$

m) $x^2\neq 4:\ \sqrt{\frac{-2+x}{(x+2)\cdot(x^2-4)}}$
$=\sqrt{\frac{x-2}{(x+2)\cdot(x+2)\cdot(x-2)}}$
$=\sqrt{\frac{1}{(x+2)^2}}=\frac{1}{x+2}$

Wurzelziehen (Teil 2)

a) $\sqrt{\underline{\ 144\ }}=12$

b) $\sqrt{\underline{\ 52\ }-3}=7$

c) $\sqrt{2\cdot\underline{\ 32\ }}=8$

d) $\sqrt{a^5\cdot\underline{\ a^7\ }}=a^6$

e) $\sqrt{a^2\underline{\ +8a\ }+16}=a+4$

a) $\sqrt{12n\underline{\ +36+n^2\ }}=n+6$

Teilweise Wurzelziehen

a)

Zahl	Primfaktor
63	3
21	3
7	7

$63 = 3^2\cdot 7$

$$\begin{aligned}\sqrt{63} &= \sqrt{3^2\cdot 7}\\ &= 3\cdot\sqrt{7}\\ &= 3\cdot\sqrt{7}\end{aligned}$$

b)

Zahl	Primfaktor
375	3
125	5
25	5
5	5

$375 = 3\cdot 5^3$

$$\begin{aligned}\sqrt{375} &= \sqrt{3\cdot 5^3}\\ &= \sqrt{3\cdot 5\cdot 5^2}\\ &= 5\cdot\sqrt{3\cdot 5}\\ &= 5\cdot\sqrt{15}\end{aligned}$$

c)

Zahl	Primfaktor
567	3
189	3
63	3
21	3
7	7

$567 = 3^4 \cdot 7$

$$\begin{aligned}\sqrt{567} &= \sqrt{3^4 \cdot 7} \\ &= \sqrt{\left(3^2\right)^2 \cdot 7} \\ &= 3^2 \cdot \sqrt{7} \\ &= 9 \cdot \sqrt{7}\end{aligned}$$

d)

Zahl	Primfaktor
2835	3
945	3
315	3
105	3
35	5
7	7

$2835 = 3^4 \cdot 5 \cdot 7$

$$\begin{aligned}\sqrt{2835} &= \sqrt{3^4 \cdot 5 \cdot 7} \\ &= \sqrt{\left(3^2\right)^2 \cdot 5 \cdot 7} \\ &= 3^2 \cdot \sqrt{5 \cdot 7} \\ &= 9 \cdot \sqrt{35}\end{aligned}$$

e) $\sqrt{4x} = \sqrt{2^2 x} = 2\sqrt{x}$

f) $\sqrt{a^5} = \sqrt{a^{4+1}} = \sqrt{a^4 \cdot a} = \sqrt{(a^2)^2 \cdot a} = a^2 \cdot \sqrt{a}$

g) $\sqrt{x^3 \cdot y^7} = \sqrt{x^{2+1} \cdot y^{6+1}} = \sqrt{x^2 \cdot x \cdot y^6 \cdot y}$
$= \sqrt{x^2 \cdot x \cdot (y^3)^2 \cdot y} = x \cdot y^3 \cdot \sqrt{x \cdot y}$

h) $\sqrt{a \cdot c^2 + c^2 d^2} = \sqrt{c^2 (a + d^2)} = c \cdot \sqrt{a + d^2}$

i) $\sqrt{(a-4) \cdot (a^2 - 16)} = \sqrt{(a-4) \cdot (a-4) \cdot (a+4)}$
$= \sqrt{(a-4)^2 \cdot (a+4)} = (a-4) \cdot \sqrt{a+4}$

Wurzelterme umformen

a) $a^{-\frac{1}{2}}=a^{\frac{1}{2}\cdot(-1)}=\left(a^{\frac{1}{2}}\right)^{-1}=\frac{1}{a^{\frac{1}{2}}}=\frac{1}{\sqrt{a}}$

b) $\frac{\sqrt{a}}{a}=\frac{\sqrt{a}}{(\sqrt{a})^2}=\frac{\sqrt{a}}{\sqrt{a}\cdot\sqrt{a}}=\frac{1}{\sqrt{a}}$

c) $\sqrt{a^3}=(a^3)^{\frac{1}{2}}=a^{3\cdot\frac{1}{2}}=a^{\frac{3}{2}}$

d) $\frac{a}{\sqrt{a}}=\frac{(\sqrt{a})^2}{\sqrt{a}}=\frac{\sqrt{a}\cdot\sqrt{a}}{\sqrt{a}}=\sqrt{a}$

e) $\sqrt{\frac{a}{b}}=\sqrt{\frac{a\cdot a}{a\cdot b}}=\sqrt{\frac{a^2}{a\cdot b}}=\frac{\sqrt{a^2}}{\sqrt{a\cdot b}}=\frac{a}{\sqrt{a\cdot b}}$

$=a\cdot\frac{1}{\sqrt{a\cdot b}}=a\cdot\frac{\sqrt{1}}{\sqrt{a\cdot b}}=a\cdot\sqrt{\frac{1}{a\cdot b}}$

f) $a^{\frac{1}{3}}=b \Leftrightarrow \left(a^{\frac{1}{3}}\right)^3=b^3 \underset{\left(a^{\frac{1}{3}}\right)^3=a^{\frac{1}{3}\cdot 3}=a^1=a}{\Leftrightarrow} b^3=a$