Trigonometrische Gleichungen (Lösungsvorschläge)

Gleichungen (1)

a)

$$\sin(x)=\frac{1}{\sqrt{2}} \underset{\arcsin\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}}{\Rightarrow} \begin{cases} x=\frac{\pi}{4}+n\cdot 2\pi=\frac{\pi+n\cdot 8\pi}{4}=\frac{(1+8n)\pi}{4} \\ \text{oder} \\ x=\pi-\frac{\pi}{4}+n\cdot 2\pi=\frac{3}{4}\pi+n\cdot 2\pi \\ \quad =\frac{3\pi+n\cdot 8\pi}{4}=\frac{(3+8n)\pi}{4} \end{cases}, \; n\in\mathbb{Z}$$

$$x\in\left[\frac{\pi}{2},\, \frac{5}{2}\pi\right]$$

n	0	1	2
$x=\frac{(1+8n)\pi}{4}$	$\frac{\pi}{4}$	$\frac{9}{4}\pi$	$\frac{17}{4}\pi$
$x=\frac{(3+8n)\pi}{4}$	$\frac{3}{4}\pi$	$\frac{11}{4}\pi$	$\frac{19}{4}\pi$

$$\Rightarrow\; L=\left\{\frac{3}{4}\pi,\, \frac{9}{4}\pi\right\}$$

b)

$$\cos(x)=\frac{\sqrt{3}}{2} \underset{\arccos\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{6}}{\Rightarrow} \begin{cases} x=\frac{\pi}{6}+n\cdot 2\pi=\frac{\pi+n\cdot 12\pi}{6}=\frac{(1+12n)\pi}{6} \\ \text{oder} \\ x=2\pi-\frac{\pi}{6}+n\cdot 2\pi=\frac{11}{6}\pi+n\cdot 2\pi \\ \quad =\frac{11\pi+n\cdot 12\pi}{6}=\frac{(11+12n)\pi}{6} \end{cases}, \; n\in\mathbb{Z}$$

$$x\in\left[-2\pi,\, \frac{\pi}{2}\right]$$

n	–2	–1	0	1
$x=\frac{(1+12n)\pi}{6}$	$-\frac{23}{6}\pi$	$-\frac{11}{6}\pi$	$\frac{\pi}{6}$	$\frac{23}{6}\pi$
$x=\frac{(11+12n)\pi}{6}$	$-\frac{13}{6}\pi$	$-\frac{\pi}{6}$	$\frac{11}{6}\pi$	$\frac{23}{6}\pi$

$$\Rightarrow\; L=\left\{-\frac{11}{6}\pi,\, -\frac{\pi}{6},\, \frac{\pi}{6}\right\}$$

c)

$$\cos(x)=-0{,}2273 \underset{\arccos(-0{,}2273)\approx 1{,}8}{\Rightarrow} \begin{cases} x=1{,}8+n\cdot 2\pi \\ \text{oder} \\ x=2\pi-1{,}8+n\cdot 2\pi\approx 4{,}48+n\cdot 2\pi \end{cases},\ n\in\mathbb{Z}$$

$x\in[6,\ 25]$

n	0	1	2	3	4
$x=1{,}8+n\cdot 2\pi$	1,8	8,08	14,37	20,65	26,93
$x=4{,}48+n\cdot 2\pi$	4,48	10,76	17,05	23,33	29,61

$\Rightarrow\ L=\{8{,}08\ ,\ 10{,}76\ ,\ 14{,}37\ ,\ 17{,}05\ ,\ 20{,}65\ ,\ 23{,}33\}$

d)

$$\sin(x)=0{,}5647 \underset{\arcsin(0{,}5647)\approx 0{,}6}{\Rightarrow} \begin{cases} x=0{,}6+n\cdot 2\pi \\ \text{oder} \\ x=\pi-0{,}6+n\cdot 2\pi\approx 2{,}54+n\cdot 2\pi \end{cases},\ n\in\mathbb{Z}$$

$x\in\mathbb{R}$

$\Rightarrow\ L=\{0{,}6+n\cdot 2\pi\ \vee\ 2{,}54+n\cdot 2\pi \mid n\in\mathbb{Z}\}$

Gleichungen (2)

a)

$$\begin{aligned} 2\sin(x)+5 &= 4 && \mid -5 \\ 2\sin(x) &= -1 && \mid \div 2 \\ \sin(x) &= -\frac{1}{2} \end{aligned}$$

$$\sin(x)=-\frac{1}{2} \underset{\arcsin\left(-\frac{1}{2}\right)=-\frac{\pi}{6}}{\Rightarrow} \begin{cases} x=-\frac{\pi}{6}+n\cdot 2\pi=\frac{n\cdot 12\pi-\pi}{6}=\frac{(12n-1)\pi}{6} \\ \text{oder} \\ x=-\pi-\left(-\frac{\pi}{6}\right)+n\cdot 2\pi=-\frac{5}{6}\pi+n\cdot 2\pi \\ \quad =\frac{n\cdot 12\pi-5\pi}{6}=\frac{(12n-5)\pi}{6} \end{cases},\ n\in\mathbb{Z}$$

$x\in[0,\ 2\pi]$

n	0	1	2
$x=\frac{(12n-1)\pi}{6}$	$-\frac{\pi}{6}$	$\frac{11}{6}\pi$	$\frac{23}{6}\pi$
$x=\frac{(12n-5)\pi}{6}$	$-\frac{5}{6}\pi$	$\frac{7}{6}\pi$	$\frac{19}{6}\pi$

$\Rightarrow\ L=\left\{\frac{7}{6}\pi,\ \frac{11}{6}\pi\right\}$

b) $$\begin{aligned} \frac{1}{2}\cos(x)+\frac{3}{2} &= 1 && |\cdot 2 \\ \cos(x)+3 &= 2 && |-3 \\ \cos(x) &= -1 \end{aligned}$$

$$\cos(x)=-1 \underset{\arccos(-1)=\pi}{\Rightarrow} x=\pi+n\cdot 2\pi=(1+2n)\pi,\ n\in\mathbb{Z}$$

$x\in[12,\ 23]$

n	0	1	2	3	4
$x=(1+2n)\pi$	π	3π	5π	7π	9π

$\Rightarrow\ L=\{5\pi,\ 7\pi\}$

c) $$\begin{aligned} \sin(x) &= \sqrt{3}-\sin(x) && |+\sin(x) \\ 2\sin(x) &= \sqrt{3} && |\div 2 \\ \sin(x) &= \frac{\sqrt{3}}{2} \end{aligned}$$

$$\sin(x)=\frac{\sqrt{3}}{2} \underset{\arcsin\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{3}}{\Rightarrow} \begin{cases} x=\frac{\pi}{3}+n\cdot 2\pi=\frac{\pi+n\cdot 6\pi}{3}=\frac{(1+6n)\pi}{3} \\ \text{oder} \\ x=\pi-\frac{\pi}{3}+n\cdot 2\pi=\frac{2}{3}\pi+n\cdot 2\pi \\ \quad =\frac{2\pi+n\cdot 6\pi}{3}=\frac{(2+6n)\pi}{3} \end{cases},\ n\in\mathbb{Z}$$

$x\in\left[-\frac{3}{2}\pi,\ \frac{5}{2}\pi\right]$

n	–1	0	1
$x=\frac{(1+6n)\pi}{3}$	$-\frac{5}{3}\pi$	$\frac{\pi}{3}$	$\frac{7}{3}\pi$
$x=\frac{(2+6n)\pi}{3}$	$-\frac{4}{3}\pi$	$\frac{2}{3}\pi$	$\frac{8}{3}\pi$

$\Rightarrow\ L=\left\{-\frac{4}{3}\pi,\ \frac{\pi}{3},\ \frac{2}{3}\pi,\ \frac{7}{3}\pi\right\}$

d)

$$\begin{aligned} 26\cos(x) &= \frac{27}{2}+\cos(x) && \mid \cdot 2 \\ 52\cos(x) &= 27+2\cos(x) && \mid -2\cos(x) \\ 50\cos(x) &= 27 && \mid \div 50 \\ \cos(x) &= \frac{27}{50} \end{aligned}$$

$$\cos(x)=\frac{27}{50} \underset{\arccos\left(\frac{27}{50}\right)\approx 1}{\Rightarrow} \begin{cases} x=1+n\cdot 2\pi \\ \text{oder} \\ x=2\pi-1+n\cdot 2\pi \approx 5{,}283+n\cdot 2\pi \end{cases}, \ n\in\mathbb{Z}$$

$x\in[0{,}5 \ , \ 15]$

n	–1	0	1	2
$x=1+n\cdot 2\pi$	–5,283	1	7,283	13,566
$x=5{,}283+n\cdot 2\pi$	–1	5,283	11,566	17,849

$\Rightarrow \ L=\{1 \ , \ 5{,}283 \ , \ 7{,}283 \ , \ 11{,}566 \ , \ 13{,}566\}$

e)

$$\begin{aligned} 21\left(\frac{1}{5}+\cos(x)\right) &= 11\cos(x) && \mid \text{Klammern auflösen} \\ \frac{21}{5}+21\cos(x) &= 11\cos(x) && \mid -21\cos(x) \\ \frac{21}{5} &= -10\cos(x) && \mid \div(-10) \\ \cos(x) &= -\frac{21}{50} \end{aligned}$$

$$\cos(x)=-\frac{21}{50} \underset{\arccos\left(-\frac{21}{50}\right)\approx 2}{\Rightarrow} \begin{cases} x=2+n\cdot 2\pi \\ \text{oder} \\ x=2\pi-2+n\cdot 2\pi \approx 4{,}283+n\cdot 2\pi \end{cases}, \ n\in\mathbb{Z}$$

$x\in[6\pi, \ 9\pi] \quad (6\pi\approx 18{,}85 \ \wedge \ 9\pi\approx 28{,}27)$

n	0	1	2	3	4
$x=2+n\cdot 2\pi$	2	8,283	14,566	20,85	27,133
$x=4{,}283+n\cdot 2\pi$	4,283	10,566	16,849	23,133	29,416

$\Rightarrow \ L=\{20{,}85 \ , \ 23{,}133 \ , \ 27{,}133\}$

f)

$$\begin{aligned} 2 &= \frac{8{,}9067}{\cos(x)+2}-1 && |+1 \\ 3 &= \frac{8{,}9067}{\cos(x)+2} && |\cdot(\cos(x)+2) \\ 3(\cos(x)+2) &= 8{,}9067 && |\text{Klammern auflösen} \\ 3\cos(x)+6 &= 8{,}9067 && |-6 \\ 3\cos(x) &= 2{,}9067 && |\div 3 \\ \cos(x) &= 0{,}9689 \end{aligned}$$

$$\cos(x)=0{,}9689 \underset{\arccos(0{,}9689)\approx 0{,}25}{\Rightarrow} \begin{cases} x=0{,}25+n\cdot 2\pi \\ \text{oder} \\ x=2\pi-0{,}25+n\cdot 2\pi \approx 6{,}033+n\cdot 2\pi \end{cases},\ n\in\mathbb{Z}$$

$$x\in\left[-\frac{5}{2}\pi,\ \frac{\pi}{2}\right] \quad \left(-\frac{5}{2}\approx -7{,}85 \ \wedge\ \frac{\pi}{2}\approx 1{,}57\right)$$

n	–2	–1	0	1
$x=0{,}25+n\cdot 2\pi$	–12,316	–6,033	0,25	6,533
$x=6{,}033+n\cdot 2\pi$	–6,533	–0,25	6,033	12,316

$$\Rightarrow \ L=\{-6{,}533\ ,\ -6{,}033\ ,\ -0{,}25\ ,\ 0{,}25\}$$

Gleichungen (3)

a) $$\cos\left(\pi\left(x-\frac{1}{3}\right)\right) = -\frac{1}{2} \quad \Big| \text{substituiere: } \pi\left(x-\frac{1}{3}\right)\rightarrow u$$

$$\cos(u) = -\frac{1}{2} \underset{\arccos\left(-\frac{1}{2}\right)=\frac{2}{3}\pi}{\Rightarrow} \begin{cases} u=\frac{2}{3}\pi+n\cdot 2\pi=\frac{2\pi+n\cdot 6\pi}{3}=\frac{(2+6n)\pi}{3} \\ \text{oder} \\ u=2\pi-\frac{2}{3}\pi+n\cdot 2\pi=\frac{4}{3}\pi+n\cdot 2\pi \\ \quad =\frac{4\pi+n\cdot 6\pi}{3}=\frac{(4+6n)\pi}{3} \end{cases}, \; n\in\mathbb{Z}$$

Rücksubstitution:

$$u\rightarrow\pi\left(x-\frac{1}{3}\right)$$

$$\begin{aligned} \pi\left(x-\frac{1}{3}\right) &= \frac{(2+6n)\pi}{3} && |\div\pi \\ x-\frac{1}{3} &= \frac{2+6n}{3} && |\cdot 3 \\ 3x-1 &= 2+6n && |+1 \\ 3x &= 3+6n && |\div 3 \\ x &= 1+2n \end{aligned}$$

$$\begin{aligned} \pi\left(x-\frac{1}{3}\right) &= \frac{(4+6n)\pi}{3} && |\div\pi \\ x-\frac{1}{3} &= \frac{4+6n}{3} && |\cdot 3 \\ 3x-1 &= 4+6n && |+1 \\ 3x &= 5+6n && |\div 3 \\ x &= \frac{5+6n}{3} \end{aligned}$$

$$x\in\left[-\frac{1}{2}, 3\right]$$

n	–1	0	1
$x=1+2n$	–1	1	3
$x=\frac{(5+6n)}{3}$	$-\frac{1}{3}$	$\frac{5}{3}$	$\frac{11}{3}$

$$\Rightarrow \; L=\left\{-\frac{1}{3}\;,\;1\;,\;\frac{5}{3}\;,\;3\right\}$$

b) $\sin(\pi x) = 0{,}5878$ | substituiere: $\pi x \to u$

$$\sin(u) = 0{,}5878 \underset{\arcsin(0{,}5878)\approx 0{,}6383\approx\frac{\pi}{5}}{\Rightarrow} \begin{cases} x=\frac{\pi}{5}+n\cdot 2\pi=\frac{\pi+n\cdot 10\pi}{5}=\frac{(1+10n)\pi}{5} \\ \text{oder} \\ x=\pi-\frac{\pi}{5}+n\cdot 2\pi=\frac{4}{5}\pi+n\cdot 2\pi \\ \quad =\frac{4\pi+n\cdot 10\pi}{5}=\frac{(4+10n)\pi}{5} \end{cases}, \; n\in\mathbb{Z}$$

Rücksubstitution:

$u \to \pi x$

$$\pi x = \frac{(1+10n)\pi}{5} \quad |\div\pi$$

$$x = \frac{1+10n}{5}$$

$$\pi x = \frac{(4+10n)\pi}{5} \quad |\div\pi$$

$$x = \frac{4+10n}{5}$$

$$x\in\left[-\frac{\pi}{2},\frac{2}{3}\pi\right] \quad \left(-\frac{\pi}{2}\approx -1{,}57 \;\wedge\; \frac{2}{3}\pi\approx 2{,}1\right)$$

n	–1	0	1
$x=\frac{1+10n}{5}$	$-\frac{9}{5}$	$\frac{1}{5}$	$\frac{11}{5}$
$x=\frac{4+10n}{5}$	$-\frac{6}{5}$	$\frac{4}{5}$	$\frac{14}{5}$

$$\Rightarrow \; L=\left\{-\frac{6}{5},\frac{1}{5},\frac{4}{5}\right\}$$

c) $\sin\left(\frac{\pi}{2}(4x-1)\right) = -1 \quad \Big|$ substituiere: $\frac{\pi}{2}(4x-1) \rightarrow u$

$$\sin(u) = -1 \underset{\arcsin(-1)=-\frac{\pi}{2}}{\Rightarrow} x = -\frac{\pi}{2}+n\cdot 2\pi = \frac{n\cdot 4\pi-\pi}{2} = \frac{(4n-1)\pi}{2},\ n\in\mathbb{Z}$$

Rücksubstitution: $u \rightarrow \frac{\pi}{2}(4x-1)$

Für $n\in\mathbb{Z}$ ist:

$$\begin{aligned} \frac{\pi}{2}(4x-1) &= \frac{4n-1}{2}\pi && |\cdot 2 \\ \pi(4x-1) &= (4n-1)\pi && |\div\pi \\ 4x-1 &= 4n-1 && |+1 \\ 4x &= 4n && |\div 4 \\ x &= n \end{aligned}$$

$\Rightarrow \quad L=\{12\ ,\ 13\ ,\ 14\}$

d) $$\begin{aligned} \frac{2}{3}\cos\left(x+\frac{5}{6}\pi\right) &= -\frac{1}{\sqrt{3}} && |\cdot 3 \\ 2\cos\left(x+\frac{5}{6}\pi\right) &= -\sqrt{3} && |\div 2 \\ \cos\left(x+\frac{5}{6}\pi\right) &= -\frac{\sqrt{3}}{2} && \Big| \text{substituiere: } x+\frac{5}{6}\pi \rightarrow u \end{aligned}$$

$$\cos(u) = -\frac{\sqrt{3}}{2} \underset{\arccos\left(-\frac{\sqrt{3}}{2}\right)=\frac{5}{6}\pi}{\Rightarrow} \begin{cases} u=\frac{5}{6}\pi+n\cdot 2\pi=\frac{5\pi+n\cdot 12\pi}{6}=\frac{(5+12n)\pi}{6} \\ \text{oder} \\ u=2\pi-\frac{5}{6}\pi+n\cdot 2\pi=\frac{7}{6}\pi+n\cdot 2\pi \\ \quad =\frac{7\pi+n\cdot 6\pi}{6}=\frac{(7+6n)\pi}{6} \end{cases},\ n\in\mathbb{Z}$$

Rücksubstitution:

$u \rightarrow x+\frac{5}{6}\pi$

$$\begin{aligned} x+\frac{5}{6}\pi &= \frac{(5+12n)\pi}{6} && |\cdot 6 \\ 6x+5\pi &= 5\pi+12n\pi && |-5\pi \\ 6x &= 12n\pi && |\div 6 \\ x &= 2n\pi \end{aligned}$$

$$\begin{aligned} x+\frac{5}{6}\pi &= \frac{(7+12n)\pi}{6} && |\cdot 6 \\ 6x+5\pi &= 7\pi+12n\pi && |-5\pi \\ 6x &= 2\pi+12n\pi && |\div 6 \\ x &= \frac{1}{3}\pi+2n\pi \\ &= \frac{(1+6n)\pi}{3} \end{aligned}$$

$$x \in \left[-\frac{7}{4}\pi\ ,\ \frac{9}{4}\pi \right]$$

n	–1	0	1
$x=2n\pi$	-2π	0	2π
$x=\frac{(1+6n)\pi}{3}$	$-\frac{5}{3}\pi$	$\frac{1}{3}\pi$	$\frac{7}{3}\pi$

$$\Rightarrow\ L=\left\{ -\frac{5}{3}\pi\ ,\ 0\ ,\ \frac{1}{3}\pi\ ,\ 2\pi \right\}$$

e)
$$\begin{aligned}
2\sin\left(\frac{\pi}{4}x+\pi\right)+5 &= 6+\sin\left(\frac{\pi}{4}x+\pi\right) && \Big|-5 \\
2\sin\left(\frac{\pi}{4}x+\pi\right) &= 1+\sin\left(\frac{\pi}{4}x+\pi\right) && \Big|-\sin\left(\frac{\pi}{4}x+\pi\right) \\
\sin\left(\frac{\pi}{4}x+\pi\right) &= 1 && \Big|\text{substituiere: } \frac{\pi}{4}x+\pi \rightarrow u \\
\sin(u) &= 1 && \underset{\arcsin(1)=\frac{\pi}{2}}{\Rightarrow}\ u=\frac{\pi}{2}+n\cdot 2\pi=\frac{\pi+n\cdot 4\pi}{2}=\frac{(1+4n)\pi}{2}\ ,\ n\in\mathbb{Z}
\end{aligned}$$

Rücksubstitution: $u \rightarrow \frac{\pi}{4}x+\pi$

$$\begin{aligned}
\frac{\pi}{4}x+\pi &= \frac{1+4n}{2}\pi && \Big|\cdot 4 \\
\pi x+4\pi &= (2+8n)\pi && \Big|\div\pi \\
x+4 &= 2+8n && \Big|-4 \\
x &= -2+8n
\end{aligned}$$

$0<x<22$

n	0	1	2	3
$x=-2+8n$	–2	6	14	22

$$\Rightarrow L\ \{6\ ,\ 14\}$$

f)
$$\begin{aligned}
4\sin\left(\frac{2x-3\pi}{18}\right)+\cos\left(\frac{x}{18}\right) &= \cos\left(\frac{x}{18}\right)-2 && \Big|-\cos\left(\frac{x}{18}\right) \\
4\sin\left(\frac{2x-3\pi}{18}\right) &= -2 && \Big|\div 4 \\
\sin\left(\frac{2x-3\pi}{18}\right) &= -\frac{1}{2} && \Big|\text{substituiere: } \frac{2x-3\pi}{18}\rightarrow u
\end{aligned}$$

$$\sin(u)=-\frac{1}{2}\ \underset{\arcsin\left(-\frac{1}{2}\right)=-\frac{\pi}{6}}{\Rightarrow}\ \begin{cases} u=-\frac{\pi}{6}+n\cdot 2\pi=\frac{n\cdot 12\pi-\pi}{6}=\frac{(12n-1)\pi}{6} \\ \text{oder} \\ u=-\pi-\left(-\frac{\pi}{6}\right)+n\cdot 2\pi=-\frac{5}{6}\pi+n\cdot 2\pi \\ \quad =\frac{n\cdot 12\pi-5\pi}{6}=\frac{(12n-5)\pi}{6} \end{cases},\ n\in\mathbb{Z}$$

Rücksubstitution:

$$u\rightarrow\frac{2x-3\pi}{18}$$

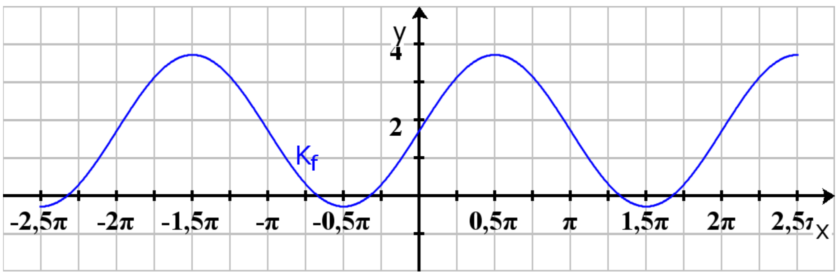
$$\begin{array}{rcll} \frac{2x-3\pi}{18} & = & \frac{(12n-1)\pi}{6} & |\cdot 18 \\ 2x-3\pi & = & 36n\pi-3\pi & |+3\pi \\ 2x & = & 36n\pi & |\div 2 \\ x & = & 18n\pi & \end{array} \qquad \begin{array}{rcll} \frac{2x-3\pi}{18} & = & \frac{(12n-5)\pi}{6} & |\cdot 18 \\ 2x-3\pi & = & 36n\pi-15\pi & |+3\pi \\ 2x & = & 18n\pi-12\pi & |\div 2 \\ x & = & 12n\pi-6\pi & \\ & = & 6\pi(2n-1) & \end{array}$$

$$\Rightarrow\ L=\{18n\pi\ \vee\ 6\pi(2n-1)\,|\,n\in\mathbb{Z}\}$$

Aufgaben aus der Analysis

1)

a)



b) Schnittpunkt mit der y-Achse: $f(0)=2\sin(0)+\sqrt{3}=\sqrt{3}\ \Rightarrow\ S_y=(0|\sqrt{3})$

Schnittpunkte mit der x-Achse:

Setze $f(x)=0$:

$$\begin{array}{rcll} 2\sin(x)+\sqrt{3} & = & 0 & |-\sqrt{3} \\ 2\sin(x) & = & -\sqrt{3} & |\div 2 \end{array}$$

$$\sin(x)\ =\ -\frac{\sqrt{3}}{2}\ \underset{\arcsin\left(-\frac{\sqrt{3}}{2}\right)=-\frac{\pi}{3}}{\Rightarrow}\ \begin{cases} x=-\frac{\pi}{3}+n\cdot 2\pi=\frac{n\cdot 6\pi-\pi}{3}=\frac{(6n-1)\pi}{3} \\ \text{oder} \\ x=-\pi-\left(-\frac{\pi}{3}\right)+n\cdot 2\pi=-\frac{2}{3}\pi+n\cdot 2\pi \\ \quad =\frac{n\cdot 6-2\pi\pi}{3}=\frac{(6n-2)\pi}{3} \end{cases},\ n\in\mathbb{Z}$$

$x \in \left[-\frac{5}{2}\pi\ ,\ \frac{5}{2}\pi\right]$

n	–2	–1	0	1	2
$x=\frac{(6n-1)\pi}{3}$	$-\frac{13}{3}\pi$	$-\frac{7}{3}\pi$	$-\frac{\pi}{3}$	$\frac{5}{3}\pi$	$\frac{11}{3}\pi$
$x=\frac{(6n-2)\pi}{3}$	$-\frac{14}{3}\pi$	$-\frac{8}{3}\pi$	$-\frac{2}{3}\pi$	$\frac{4}{3}\pi$	$\frac{10}{3}\pi$

$$\Rightarrow\ N_1=\left(-\frac{7}{3}\pi\middle|0\right)\ ,\ N_2=\left(-\frac{2}{3}\pi\middle|0\right)\ ,$$
$$N_3=\left(-\frac{\pi}{3}\middle|0\right)\ ,\ N_4=\left(\frac{5}{3}\pi\middle|0\right)\ ,\ N_5=\left(\frac{4}{3}\pi\middle|0\right)$$

c) Setze $f(x)=h(x)$:

$$\begin{aligned}
2\sin(x)+\sqrt{3} &= 3-4\sin(x)+\sqrt{3} && \left|-\sqrt{3}\right.\\
2\sin(x) &= 3-4\sin(x) && \left|+4\sin(x)\right.\\
6\sin(x) &= 3 && \left|\div 6\right.\\
\sin(x) &= \frac{1}{2}
\end{aligned}$$

$$\underset{\arcsin\left(\frac{1}{2}\right)=\frac{\pi}{6}}{\Rightarrow}\begin{cases} x=\frac{\pi}{6}+n\cdot 2\pi=\frac{\pi+n\cdot 12\pi}{6}=\frac{(1+12n)\pi}{6} \\ \text{oder} \\ x=\pi-\frac{\pi}{6}+n\cdot 2\pi=\frac{5}{6}\pi+n\cdot 2\pi \\ \quad =\frac{5\pi+n\cdot 12\pi}{6}=\frac{(5+12n)\pi}{6} \end{cases},\ n\in\mathbb{Z}$$

$x \in \left[-\frac{5}{2}\pi\ ,\ \frac{5}{2}\pi\right]$

n	–2	–1	0	1
$x=\frac{(1+12n)\pi}{6}$	$-\frac{23}{6}\pi$	$-\frac{11}{6}\pi$	$\frac{\pi}{6}$	$\frac{13}{6}\pi$
$x=\frac{(5+12n)\pi}{6}$	$-\frac{19}{6}\pi$	$-\frac{7}{6}\pi$	$\frac{5}{6}\pi$	$\frac{17}{6}\pi$

$$x=-\frac{11}{6}\pi \quad \Rightarrow \quad f\left(-\frac{11}{6}\pi\right)=1+\sqrt{3} \quad \Rightarrow \quad S_1=\left(-\frac{11}{6}\pi\middle|1+\sqrt{3}\right)$$
$$x=-\frac{7}{6}\pi \quad \Rightarrow \quad f\left(-\frac{7}{6}\pi\right)=1+\sqrt{3} \quad \Rightarrow \quad S_2=\left(-\frac{7}{6}\pi\middle|1+\sqrt{3}\right)$$
$$x=\frac{\pi}{6} \quad \Rightarrow \quad f\left(\frac{\pi}{6}\right)=1+\sqrt{3} \quad \Rightarrow \quad S_3=\left(\frac{\pi}{6}\middle|1+\sqrt{3}\right)$$
$$x=\frac{5}{6}\pi \quad \Rightarrow \quad f\left(\frac{5}{6}\pi\right)=1+\sqrt{3} \quad \Rightarrow \quad S_4=\left(\frac{5}{6}\pi\middle|1+\sqrt{3}\right)$$
$$x=\frac{13}{6}\pi \quad \Rightarrow \quad f\left(\frac{13}{6}\pi\right)=1+\sqrt{3} \quad \Rightarrow \quad S_5=\left(\frac{13}{6}\pi\middle|1+\sqrt{3}\right)$$

2)

a) Setze $f(x)=0$:

$$\begin{aligned}
\sqrt{2}\cos(2x)-1 &= 0 && |+1\\
\sqrt{2}\cos(2x) &= 1 && |\div\sqrt{2}\\
\cos(2x) &= \frac{1}{\sqrt{2}} && |\text{ substituiere: } 2x\rightarrow u
\end{aligned}$$

$$\cos(u) = \frac{1}{\sqrt{2}} \underset{\arccos\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}}{\Rightarrow} \begin{cases} u=\frac{\pi}{4}+n\cdot 2\pi=\frac{\pi+n\cdot 8\pi}{4}=\frac{(1+8n)\pi}{4} \\ \text{oder} \\ u=2\pi-\frac{\pi}{4}+n\cdot 2\pi=\frac{7}{4}\pi+n\cdot 2\pi \\ \quad =\frac{7\pi+n\cdot 8\pi}{4}=\frac{(7+8n)\pi}{4} \end{cases} ,\ n\in\mathbb{Z}$$

Rücksubstitution: $u\rightarrow 2x$

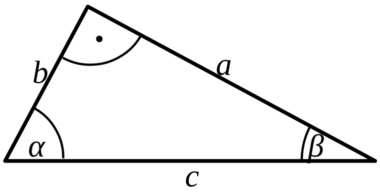
$$\begin{aligned}
2x &= \frac{(1+8n)\pi}{4} \quad |\div 2 & \qquad 2x &= \frac{(7+8n)\pi}{4} \quad |\div 2\\
x &= \frac{(1+8n)\pi}{8} & x &= \frac{(7+8n)\pi}{8}
\end{aligned}$$

$$\Rightarrow N=\left(\frac{(1+8n)\pi}{8}\middle|0\right) \vee \Rightarrow N=\left(\frac{(7+8n)\pi}{8}\middle|0\right),\quad n\in\mathbb{Z}$$

b) Setze $f(x)=h(x)$:

$$\begin{aligned}
\sqrt{2}\cos(2x)-1 &= \frac{1}{\sqrt{2}}\left(2\sin(x)-\sqrt{2}\right) && |\cdot\sqrt{2}\\
2\cos(2x)-\sqrt{2} &= 2\sin(x)-\sqrt{2} && |+\sqrt{2}\\
2\cos(2x) &= 2\sin(x) && |\div 2\\
\cos(2x) &= \sin(x)
\end{aligned}$$

In einem Rechtwinkligen Dreieck



gilt: $\sin(\alpha)=\cos(\beta)$ und $\alpha+\beta=90°=\frac{\pi}{2}$

$\Rightarrow \quad x+2x=\frac{\pi}{2} \quad \Rightarrow \quad x=\frac{\pi}{6}$

aufgrund der Periodizität ist $x=\frac{\pi}{6}+2n\pi=\frac{(1+12n)\pi}{6} \quad , \quad n\in\mathbb{Z}$

$x\in[-2\pi \ , \ 2\pi]$

n	–2	–1	0	1
$x=\frac{(1+12n)\pi}{6}$	$-\frac{23}{6}\pi$	$-\frac{11}{6}\pi$	$\frac{\pi}{6}$	$\frac{13}{6}\pi$

$x=-\frac{11}{6}\pi \quad \Rightarrow \quad f\left(-\frac{11}{6}\pi\right)=\frac{\sqrt{6}-2}{2} \quad \Rightarrow \quad S_1=\left(-\frac{11}{6}\pi \middle| \frac{\sqrt{6}-2}{2}\right)$

$x=\frac{\pi}{6} \quad \Rightarrow \quad f\left(\frac{\pi}{6}\right)=\frac{\sqrt{6}-2}{2} \quad \Rightarrow \quad S_2=\left(\frac{\pi}{6} \middle| \frac{\sqrt{6}-2}{2}\right)$